Relationship between the Mandelbrot Algorithm and the Platonic Solids

Dominic Rochon[1]
(Joint work with André Vallières)

Département de mathématiques et d'informatique
Université du Québec à Trois-Rivières
Canada

University of Hawai'i at Mānoa, USA
June 2022

[1]Research supported by CRSNG (Canada).

Tetrabrot

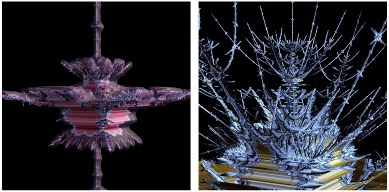


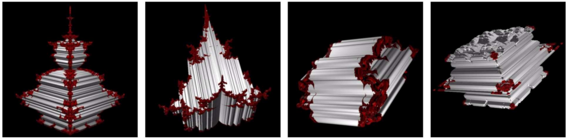
Figure 1: Deep zoom on the Tetrabrot

Quadratic polynomials iterated on hypercomplex algebras have been used to generate multidimensional Mandelbrot sets for several years. While Bedding and Briggs established that possibly no interesting dynamics occur in the case of the quaternionic Mandelbrot set, the generalization which uses the four-dimensional commutative algebra of bicomplex numbers has some interesting fractal aspects reminiscent of the classical Mandelbrot set.

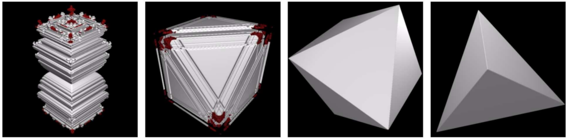
Multicomplex Extension

This bicomplex Mandelbrot set $\mathcal{M}_2$ is proven to be connected and related to a bicomplex version of the Fatou-Julia theorem. In 2009, these results and ideas were subsequently extended to the multicomplex space for quadratic polynomials of the form $z^2 + c$ over multicomplex numbers. In addition, the authors introduced an equivalence relation between the fifty-six principal 3D slices of the tricomplex Mandelbrot set $\mathcal{M}_3$ in order to establish which slices have the same dynamics and appearance in 3D visualization software.

Tricomplex Characterization



(a) $\mathcal{T}(1, \mathbf{i_1}, \mathbf{i_2})$ (b) $\mathcal{T}(1, \mathbf{i_1}, \mathbf{j_1})$ (c) $\mathcal{T}(\mathbf{i_1}, \mathbf{i_2}, \mathbf{j_1})$ (d) $\mathcal{T}(\mathbf{i_1}, \mathbf{i_2}, \mathbf{j_2})$



(e) $\mathcal{T}(\mathbf{i_1}, \mathbf{j_1}, \mathbf{j_2})$ (f) $\mathcal{T}(\mathbf{i_1}, \mathbf{i_2}, \mathbf{i_3})$ (g) $\mathcal{T}(1, \mathbf{j_1}, \mathbf{j_2})$ (h) $\mathcal{T}(\mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3})$

Figure 2: The eight principal 3D slices of $\mathcal{M}_3$

By doing this, eight equivalence classes were identified and characterized, and for each of those, one particular slice was designated as a canonical representative.

The Platonic Solids

Surprisingly, there are two principal 3D slices that exhibit no irregularity, which is in sharp contrast to the other six. In fact, one is a regular octahedron (the Airbrot), while the other is a regular tetrahedron (the Firebrot), and the underlying mechanism explaining such behavior is currently poorly understood. Thus, the main objective of this work is to deepen knowledge on the aspects mentioned above and, by extension, to establish a relationship between the Mandelbrot algorithm and the Platonic solids, which have become an integral part of the natural sciences, such as chemistry and geology, due to their remarkable properties and prevalence in those fields.

Bicomplex Numbers

Definition 1 ($\mathbb{M}(2)$ or $\mathbb{BC}$-space)

Let $z_1 = x_1 + x_2\mathbf{i_1}$, $z_2 = x_3 + x_4\mathbf{i_1}$ be two complex numbers $\mathbb{M}(1) \simeq \mathbb{C}$ with $\mathbf{i_1}^2 = -1$. A **bicomplex number** ζ is defined as:

$$\zeta = z_1 + z_2\mathbf{i_2}$$

where $\mathbf{i_2}^2 = -1$.

Various representations:

- In terms of four real numbers: $\zeta = x_1 + x_2\mathbf{i_1} + x_3\mathbf{i_2} + x_4\mathbf{j_1}$
- In terms of two idempotent elements:

$$\zeta = (z_1 - z_2\mathbf{i_1})\gamma_1 + (z_1 + z_2\mathbf{i_1})\overline{\gamma}_1$$

where $\gamma_1 = \frac{1+\mathbf{j_1}}{2}$ and $\overline{\gamma}_1 = \frac{1-\mathbf{j_1}}{2}$.

Operations on Bicomplex Numbers

Let $\zeta_1 = z_1 + z_2\mathbf{i_2}$ and $\zeta_2 = z_3 + z_4\mathbf{i_2}$.

1) Equality: $\zeta_1 = \zeta_2 \iff z_1 = z_3$ and $z_2 = z_4$
2) Addition: $\zeta_1 + \zeta_2 := (z_1 + z_3) + (z_2 + z_4)\mathbf{i_2}$
3) Multiplication: $\zeta_1 \cdot \zeta_2 := (z_1 z_3 - z_2 z_4) + (z_2 z_3 + z_1 z_4)\mathbf{i_2}$
4) Euclidean Norm: $|\zeta_1| = \sqrt{|z_1|^2 + |z_2|^2} = \sqrt{\sum_{i=1}^{4} x_i^2}$

Remark:

- $(\mathbb{M}(2), +, \cdot)$ forms a commutative ring with unity and zero divisors.
- $(\mathbb{M}(2), +, \cdot, |\cdot|)$ forms a **Banach space**.

Tricomplex Numbers

Definition 2 ($\mathbb{M}(3)$ or $\mathbb{TC}$-space)

Let $\zeta_1 = z_1 + z_2\mathbf{i_2}$, $\zeta_2 = z_3 + z_4\mathbf{i_2}$ be two bicomplex numbers. A **tricomplex number** η is defined as:

$$\eta = \zeta_1 + \zeta_2\mathbf{i_3}$$

where $\mathbf{i_3}^2 = -1$.

Various representations:

- In terms of four complex numbers: $\eta = z_1 + z_2\mathbf{i_2} + z_3\mathbf{i_3} + z_4\mathbf{j_3}$
- In terms of eight real numbers:

$$\eta = x_1 + x_2\mathbf{i_1} + x_3\mathbf{i_2} + x_4\mathbf{i_3} + x_5\mathbf{i_4} + x_6\mathbf{j_1} + x_7\mathbf{j_2} + x_8\mathbf{j_3}$$

Go to Table

Tricomplex Numbers

Various representations (continuing):

- In terms of two idempotent elements:

$$\eta = (\zeta_1 - \zeta_2\mathbf{i_2})\gamma_3 + (\zeta_1 + \zeta_2\mathbf{i_2})\overline{\gamma}_3$$

where $\zeta_1, \zeta_2 \in \mathbb{M}(2)$, $\gamma_3 = \frac{1+\mathbf{j_3}}{2}$ and $\overline{\gamma}_3 = \frac{1-\mathbf{j_3}}{2}$.

- In terms of four idempotent elements:

$$\eta = \eta_{\gamma_1\gamma_3} \cdot \gamma_1\gamma_3 + \eta_{\gamma_1\overline{\gamma}_3} \cdot \gamma_1\overline{\gamma}_3 + \eta_{\overline{\gamma}_1\gamma_3} \cdot \overline{\gamma}_1\gamma_3 + \eta_{\overline{\gamma}_1\overline{\gamma}_3} \cdot \overline{\gamma}_1\overline{\gamma}_3$$

where $\eta_{\gamma_1\gamma_3}, \eta_{\gamma_1\overline{\gamma}_3}, \eta_{\overline{\gamma}_1\gamma_3}, \eta_{\overline{\gamma}_1\overline{\gamma}_3} \in \mathbb{M}(1) \simeq \mathbb{C}$ are defined as the **projections** in the plane.

Subsets of $\mathbb{M}(3)$

Definition 3

Let $\mathbf{i_k} \in \{\mathbf{i_1}, \mathbf{i_2}, \mathbf{i_3}, \mathbf{i_4}\}$ and $\mathbf{j_k} \in \{\mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3}\}$, where $\mathbf{i_k}^2 = -\mathbf{1}$ and $\mathbf{j_k}^2 = \mathbf{1}$. We define

$$\mathbb{C}(\mathbf{i_k}) := \{x_0 + x_1\mathbf{i_k} \ : \ x_0, x_1 \in \mathbb{R}\}$$

and

$$\mathbb{D}(\mathbf{j_k}) := \{x_0 + x_1\mathbf{j_k} \ : \ x_0, x_1 \in \mathbb{R}\}.$$

- $\mathbb{C}(\mathbf{i_k})$ is a subset of $\mathbb{M}(3)$ for $k \in \{1, 2, 3, 4\}$. They are all isomorphic to $\mathbb{C}$. Notice that $\mathbb{C}(\mathbf{i_1}) = \mathbb{M}(1)$.
- $\mathbb{D}(\mathbf{j_k})$ is a subset of $\mathbb{M}(3)$ and is isomorphic to the set of hyperbolic numbers $\mathbb{D}$ for $k \in \{1, 2, 3\}$.

Subsets of $\mathbb{M}(3)$ (continuing)

Definition 4

Let $\mathbf{i_k}, \mathbf{i_l}, \mathbf{i_m} \in \{1, \mathbf{i_1}, \mathbf{i_2}, \mathbf{i_3}, \mathbf{i_4}, \mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3}\}$ with $\mathbf{i_k} \neq \mathbf{i_l}$, $\mathbf{i_k} \neq \mathbf{i_m}$ and $\mathbf{i_l} \neq \mathbf{i_m}$. The third subset is

$$\mathbb{T}(\mathbf{i_m}, \mathbf{i_k}, \mathbf{i_l}) := \{x_1\mathbf{i_m} + x_2\mathbf{i_k} + x_3\mathbf{i_l} \ : \ x_1, x_2, x_3 \in \mathbb{R}\}.$$

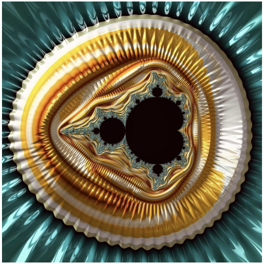
- $\mathbb{T}(\mathbf{i_m}, \mathbf{i_k}, \mathbf{i_l}) = \mathrm{span}_{\mathbb{R}}\{\mathbf{i_m}, \mathbf{i_k}, \mathbf{i_l}\}$.
- This sub-vector space of $\mathbb{M}(3)$ is used to make 3D slices in the tricomplex Mandelbrot set.

The Mandelbrot Set

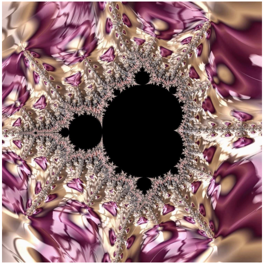
Definition 5

Let $Q_c(z) = z^2 + c$ a quadratic complex polynomial. The Mandelbrot set is defined as follows:

$$\mathcal{M}^2 = \{c \in \mathbb{C} : \{Q_c^m(0)\}_{m=1}^{\infty} \text{ is bounded } \}.$$



(a) $\mathcal{M}^2$: Mandelbrot set



(b) $\mathcal{M}^2$: Zoom in

The Hyperbrot Set

In 1990, P. Senn suggested to define the Mandelbrot set for the hyperbolic numbers.

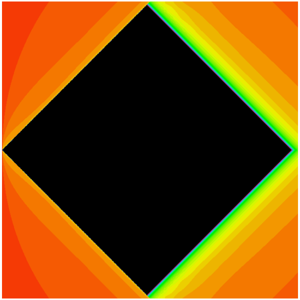
Definition 6

Let $Q_c(z) = z^2 + c$ a quadratic hyperbolic polynomial. The *Hyperbrot set* is defined as follows:

$$\mathcal{H}^2 = \left\{c \in \mathbb{D} : \{Q_c^m(0)\}_{m=1}^{\infty} \text{ is bounded } \right\}.$$

Seen noticed that the Mandelbrot set for this number structure seemed to be a square. Four years later, a proof of this statement was giving by W. Metzler.

The Hyperbolic Mandelbrot Set



(a) $\mathcal{H}^2$: Hyperbrot Set

This 2D phenomenon is the fundamental basic tool to obtain polyhedrons with some hypercomplex dynamical systems.

Tricomplex Mandelbrot Set

Definition 7

Let $Q_c(\eta) = \eta^2 + c$ where $\eta, c \in \mathbb{M}(3)$. The **tricomplex Mandelbrot set** is define as the set

$$\mathcal{M}_3^2 := \{c \in \mathbb{M}(3) \ : \ \{Q_c^m(0)\}_{m=1}^{\infty} \text{ is bounded } \}.$$

Theorem 8

A tricomplex number c is in $\mathcal{M}_3^2$ if and only if $|Q_c^m(0)| \leq 2$ for all natural number $m \geq 1$.

Principal 3D slices of $\mathcal{M}_3^2$

To visualize the 8D tricomplex Mandelbrot set, we have to define a **principal 3D slice of $\mathcal{M}_3^2$**.

$$\mathcal{T}^2 := \mathcal{T}^2(\mathbf{i_m}, \mathbf{i_k}, \mathbf{i_l}) = \{c \in \mathbb{T}(\mathbf{i_m}, \mathbf{i_k}, \mathbf{i_l}) \ : \ \{Q_c^m(0)\}_{m=1}^{\infty} \text{ is bounded } \}.$$

- There are 56 possible principal 3D slices.
- Any principal 3D slice of the **multicomplex** Mandelbrot set is equivalent to at least one quadricomplex slice or directly to one tricomplex slice up to an affine transformation. Hence, the **tricomplex space** is, in a way, optimal.

Principal Slices of $\mathcal{M}_3^2$

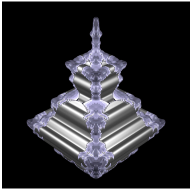
The number of principal 3D slices of $\mathcal{M}_3^2$ can be reduced to 8 slices.

Theorem 9

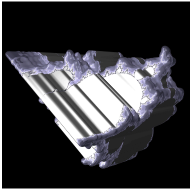
*There are eight **principal 3D slices** of the tricomplex Mandelbrot set $\mathcal{M}_3^2$:*

- $\mathcal{T}^2(1, \mathbf{i_1}, \mathbf{i_2})$ *called Tetrabrot;*
- $\mathcal{T}^2(1, \mathbf{i_1}, \mathbf{j_1})$ *called Arrowheadbrot;*
- $\mathcal{T}^2(\mathbf{i_1}, \mathbf{i_2}, \mathbf{j_1})$ *called Mousebrot;*
- $\mathcal{T}^2(\mathbf{i_1}, \mathbf{i_2}, \mathbf{j_2})$ *called Turtlebrot;*
- $\mathcal{T}^2(\mathbf{i_1}, \mathbf{j_1}, \mathbf{j_2})$ *called Hourglassbrot;*
- $\mathcal{T}^2(\mathbf{i_1}, \mathbf{i_2}, \mathbf{i_3})$ *called Metabrot;*
- $\mathcal{T}^2(1, \mathbf{j_1}, \mathbf{j_2})$ *called **Airbrot**;*
- $\mathcal{T}^2(\mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3})$ *called **Firebrot**.*

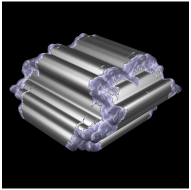
Family Shooting: $\eta^2 + c$



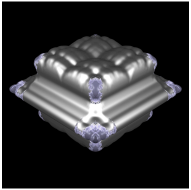
(a) Tetrabrot



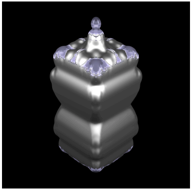
(b) Arrowheadbrot



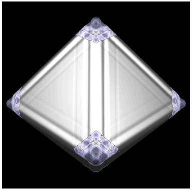
(c) Mousebrot



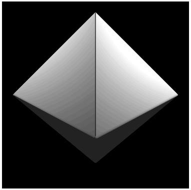
(d) Turtlebrot



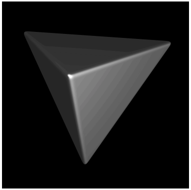
(e) Hourglassbrot



(f) Metabrot



(g) Airbrot



(h) Firebrot

Octahedron and Tetrahedron

The Airbrot is a regular octahedron of edge length equal to $\frac{9}{8}\sqrt{2}$. There is also a similar result for the Firebrot.

Theorem 10

The principal slice $\mathcal{T}(\mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3})$, named the Firebrot, can be characterized as follows:

$$\mathcal{T}(\mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3}) = \bigcup_{y \in [-\frac{1}{4}, \frac{1}{4}]} \{[(\mathcal{H}' + y\mathbf{j_1}) \cap (-\mathcal{H}' - y\mathbf{j_1})] + y\mathbf{j_2}\}$$

where $\mathcal{H}' := \{c_7\mathbf{j_3} + c_4\mathbf{j_1} : c_7 + c_4\mathbf{j_1} \in \mathcal{H}\}$.

Theorem 11

$\mathcal{T}(\mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3})$ *is a regular tetrahedron with edge length of* $\frac{\sqrt{2}}{2}$.

Biduplex Space

The main reason why the principal slices $\mathcal{T}(1, \mathbf{j_1}, \mathbf{j_2})$ and $\mathcal{T}(\mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3})$ are Platonic solids is that in both cases, the iterates calculated when applying the Mandelbrot algorithm to numbers of the form $c_1 + c_4\mathbf{j_1} + c_6\mathbf{j_2}$ and $c_4\mathbf{j_1} + c_6\mathbf{j_2} + c_7\mathbf{j_3}$ stay in a particular four-dimensional subspace of $\mathbb{TC}$ called the **biduplex numbers**. Indeed, it is easily verified that the set of biduplex numbers

$$\mathbb{D}(2) := \{c_1 + c_4\mathbf{j_1} + c_6\mathbf{j_2} + c_7\mathbf{j_3} : c_1, c_4, c_6, c_7 \in \mathbb{R} \text{ and } \mathbf{j_1}^2 = \mathbf{j_2}^2 = \mathbf{j_3}^2 = 1\}$$

together with tricomplex addition and multiplication forms a proper subring of $\mathbb{TC}$.

Biduplex Space

Real Axis

Moreover, in the right idempotent basis, every biduplex number can be expressed through four real components. It follows that the boundedness of the sequence $\{P_c^{(n)}(0)\}_{n\in\mathbb{N}}$ specific to any $c \in \mathcal{T}(1, \mathbf{j_1}, \mathbf{j_2})$ or $c \in \mathcal{T}(\mathbf{j_1}, \mathbf{j_2}, \mathbf{j_3})$ is entirely dependent on the dynamics of the classical Mandelbrot set $\mathcal{M}_1$ along the real axis, hence the regularity of these slices.

Hexahedron

This suggests another approach to generate regular polyhedra within tricomplex dynamics: to define and visualize 3D slices in a basis that is directly linked to the simple dynamics of the real line. In fact, since the set $\{\gamma_1\gamma_3, \overline{\gamma_1}\gamma_3, \gamma_1\overline{\gamma_3}, \overline{\gamma_1\gamma_3}\}$ is a basis of the vector space of $\mathbb{TC}$ with complex coefficients, the set $\{\gamma_1\gamma_3, \overline{\gamma_1}\gamma_3, \gamma_1\overline{\gamma_3}, \overline{\gamma_1\gamma_3}, \mathbf{i_1}\gamma_1\gamma_3, \mathbf{i_1}\overline{\gamma_1}\gamma_3, \mathbf{i_1}\gamma_1\overline{\gamma_3}, \mathbf{i_1}\overline{\gamma_1\gamma_3}\}$ is a basis of the same space, but with real coefficients. This brings us to the following definitions.

Definition 12

Let α, β, δ be three distinct elements taken in $\{\gamma_1\gamma_3, \overline{\gamma_1}\gamma_3, \gamma_1\overline{\gamma_3}, \overline{\gamma_1\gamma_3}, \mathbf{i_1}\gamma_1\gamma_3, \mathbf{i_1}\overline{\gamma_1}\gamma_3, \mathbf{i_1}\gamma_1\overline{\gamma_3}, \mathbf{i_1}\overline{\gamma_1\gamma_3}\}$. The space

$$\mathbb{T}(\alpha, \beta, \delta) := \text{span}_{\mathbb{R}}\{\alpha, \beta, \delta\}$$

is the vector subspace of $\mathbb{TC}$ consisting of all real finite linear combinations of these three distinct units.

Hexahedron

Definition 13

Let α, β, δ be three distinct elements taken in $\{\gamma_1\gamma_3, \overline{\gamma_1}\gamma_3, \gamma_1\overline{\gamma_3}, \overline{\gamma_1\gamma_3}, \mathbf{i_1}\gamma_1\gamma_3, \mathbf{i_1}\overline{\gamma_1}\gamma_3, \mathbf{i_1}\gamma_1\overline{\gamma_3}, \mathbf{i_1}\overline{\gamma_1\gamma_3}\}$. We define an **idempotent 3D slice** of the tricomplex Mandelbrot set $\mathcal{M}_3$ as

$$\begin{aligned}\mathcal{T}_e(\alpha,\beta,\delta) &= \{c \in \mathbb{T}(\alpha,\beta,\delta) : \{P_c^{(n)}(0)\}_{n\in\mathbb{N}} \text{ is bounded}\}\\ &= \mathbb{T}(\alpha,\beta,\delta) \cap \mathcal{M}_3.\end{aligned}$$

There are still 56 possible slices in total. For the sake of brevity, we will introduce the only 3D slice that is of interest regarding our objective. The idempotent 3D slice $\mathcal{T}_e(\gamma_1\gamma_3, \overline{\gamma_1}\gamma_3, \gamma_1\overline{\gamma_3})$, called the **Earthbrot**.

Theorem 14

The Earthbrot is a cube with edge length of $\frac{9}{4}$.

Stellated Octahedron

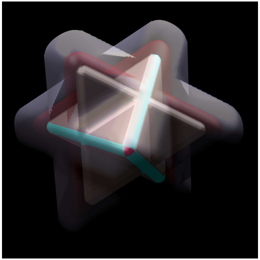


Figure 3: The stellated octahedron (also called Starbrot) with various divergence layers

It seems highly unlikely that the two remaining Platonic solids, the dodecahedron and the icosahedron, can be visualized through tricomplex dynamics. However, it is possible to generate other types of polyhedra, like regular compounds and some specific Archimedean solids. As a basic example, consider the stellated octahedron, which can be seen as a regular dual compound made of two regular tetrahedra. We can obtain the geometric dual of the slice $\mathcal{T}(\mathbf{j}_1, \mathbf{j}_2, \mathbf{j}_3)$ by using a tricomplex conjugate to apply a specific reflection to it. Thus, within tricomplex dynamics, we have a simple way to visualize a stellated octahedron as the union of the Firebrot and its dual.

Conclusion

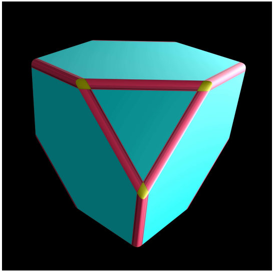


Figure 4: $\mathcal{T}(\mathbf{j}_1, \mathbf{j}_2, \mathbf{j}_3)$ power 8

In this presentation, we provided an overview of some of the important results in the field of multicomplex dynamics developed in the last 20 years. In the process, we also confirmed the presence of **three Platonic solids** within the tricomplex dynamics associated with the Mandelbrot algorithm. In subsequent works, it could be interesting to extend the geometrical study of the principal slices of $\mathcal{M}_3$ to the power $p \geq 2$. Doing so would probably expand the list of convex polyhedra found among the principal slices because when considering $p = 8$, the Firebrot strongly resembles a truncated tetrahedron, which is an Archimedean solid.

Conjecture

In addition, by considering the algebra $\mathbb{M}(n)$, $n \geq 3$, the search for regular convex polytopes could be generalised to n-dimensional slices. In fact, it is worth noting that the method used previously provides a straightforward way to establish that the idempotent 4D slice $\mathcal{T}_e(\gamma_1\gamma_3, \overline{\gamma_1}\gamma_3, \gamma_1\overline{\gamma_3}, \overline{\gamma_1\gamma_3})$ is a tesseract (also called hypercube), that is, a four-dimensional regular convex polytope. Furthermore, the approach for the Airbrot and the Firebrot can probably be used to prove that in the usual basis, at least one specific 4D slice corresponds to a regular four-dimensional cross-polytope (also called hyperoctahedron). Together, these examples provide sufficient information to allow us to emit a conjecture.

Conjecture 1

Let $n \geq 3$. The multicomplex Mandelbrot set $\mathcal{M}_n$ contains exactly three regular convex n-polytopes among all possible principal or idempotent n-dimensional slices.

Thanks for your attention!

Figure 5: This is a look at the so-called "Seahorse Valley" of the Mandelbrot set when generated with a spheroidal dynamical system.

References: www.3dfractals.com

- Baley Price, G., *An Introduction to Multicomplex Spaces and Functions*, Monographs and textbooks on pure and applied mathematics, 1991.
- Rochon, D., *A Generalized Mandelbrot Set for Bicomplex Numbers*, Fractals **8**(4), 355-368 (2000).
- Wang, X.-y. and Song W.-J., *The Genralized M-J Sets for Bicomplex Numbers*, Non linear Dynamics **72**, 17-26 (2013).
- Parisé, P.-O. and Rochon, D., *Tricomplex Dynamical Systems Generated by Polynomials of Odd Degree*, Fractals **25**(3), 1-11 (2017).
- Brouillette, G., and Rochon, D., *Characterization of the Principal 3D Slices Related to the Multicomplex Mandelbrot Set*, AACA **29**(39), (2019).
- Vallières, A., and Rochon, D., *Relationship between the Mandelbrot Algorithm and the Platonic Solids*, Mathematics **10**(482), (2022).

Table of imaginary units

$\cdot$	1	$\mathbf{i_1}$	$\mathbf{i_2}$	$\mathbf{i_3}$	$\mathbf{i_4}$	$\mathbf{j_1}$	$\mathbf{j_2}$	$\mathbf{j_3}$
1	1	$\mathbf{i_1}$	$\mathbf{i_2}$	$\mathbf{i_3}$	$\mathbf{i_4}$	$\mathbf{j_1}$	$\mathbf{j_2}$	$\mathbf{j_3}$
$\mathbf{i_1}$	$\mathbf{i_1}$	$-\mathbf{1}$	$\mathbf{j_1}$	$\mathbf{j_2}$	$-\mathbf{j_3}$	$-\mathbf{i_2}$	$-\mathbf{i_3}$	$\mathbf{i_4}$
$\mathbf{i_2}$	$\mathbf{i_2}$	$\mathbf{j_1}$	$-\mathbf{1}$	$\mathbf{j_3}$	$-\mathbf{j_2}$	$-\mathbf{i_1}$	$\mathbf{i_4}$	$-\mathbf{i_3}$
$\mathbf{i_3}$	$\mathbf{i_3}$	$\mathbf{j_2}$	$\mathbf{j_3}$	$-\mathbf{1}$	$-\mathbf{j_1}$	$\mathbf{i_4}$	$-\mathbf{i_1}$	$-\mathbf{i_2}$
$\mathbf{i_4}$	$\mathbf{i_4}$	$-\mathbf{j_3}$	$-\mathbf{j_2}$	$-\mathbf{j_1}$	$-\mathbf{1}$	$\mathbf{i_3}$	$\mathbf{i_2}$	$\mathbf{i_1}$
$\mathbf{j_1}$	$\mathbf{j_1}$	$-\mathbf{i_2}$	$-\mathbf{i_1}$	$\mathbf{i_4}$	$\mathbf{i_3}$	$\mathbf{1}$	$-\mathbf{j_3}$	$-\mathbf{j_2}$
$\mathbf{j_2}$	$\mathbf{j_2}$	$-\mathbf{i_3}$	$\mathbf{i_4}$	$-\mathbf{i_1}$	$\mathbf{i_2}$	$-\mathbf{j_3}$	$\mathbf{1}$	$-\mathbf{j_1}$
$\mathbf{j_3}$	$\mathbf{j_3}$	$\mathbf{i_4}$	$-\mathbf{i_3}$	$-\mathbf{i_2}$	$\mathbf{i_1}$	$-\mathbf{j_2}$	$-\mathbf{j_1}$	$\mathbf{1}$

Table 1: Product of tricomplex imaginary units

Back